Fiche de calcul n° 8

# Trigonométrie

007A

**Prérequis**
Relation $\cos^2 + \sin^2 = 1$. Symétrie et périodicité de sin et cos.
Formules d'addition et de duplication. Fonction tangente.

Dans toute cette fiche, $x$ désigne une quantité réelle.

## Valeurs remarquables de cosinus et sinus

### Calcul 8.1

Simplifier :

a) $\cos\dfrac{\pi}{4} + \cos\dfrac{3\pi}{4} + \cos\dfrac{5\pi}{4} + \cos\dfrac{7\pi}{4}$ .

b) $\sin\dfrac{5\pi}{6} + \sin\dfrac{7\pi}{6}$ ..................

c) $\tan\dfrac{2\pi}{3} + \tan\dfrac{3\pi}{4} + \tan\dfrac{5\pi}{6} + \tan\dfrac{7\pi}{6}$

d) $\cos^2\dfrac{4\pi}{3} - \sin^2\dfrac{4\pi}{3}$ ................

## Propriétés remarquables de cosinus et sinus

### Calcul 8.2

Simplifier :

a) $\sin(\pi - x) + \cos\left(\dfrac{\pi}{2} + x\right)$ .........

b) $\sin(-x) + \cos(\pi + x) + \sin\left(\dfrac{\pi}{2} - x\right)$

c) $\sin\left(\dfrac{\pi}{2} - x\right) + \sin\left(\dfrac{\pi}{2} + x\right)$ ........

d) $\cos(x - \pi) + \sin\left(-\dfrac{\pi}{2} - x\right)$ .......

## Formules d'addition

### Calcul 8.3

Calculer les quantités suivantes.

a) $\cos\dfrac{5\pi}{12}$ (on a $\dfrac{\pi}{6} + \dfrac{\pi}{4} = \dfrac{5\pi}{12}$) .......

b) $\cos\dfrac{\pi}{12}$ ...........................

c) $\sin\dfrac{\pi}{12}$ ............................

d) $\tan\dfrac{\pi}{12}$ ...........................

### Calcul 8.4

a) Simplifier : $\sin(4x)\cos(5x) - \sin(5x)\cos(4x)$ .................................

b) Simplifier : $\dfrac{\sin 2x}{\sin x} - \dfrac{\cos 2x}{\cos x}$ (pour $x \in \left]0, \dfrac{\pi}{2}\right[$) ................................

c) Simplifier : $\cos x + \cos\left(x + \dfrac{2\pi}{3}\right) + \cos\left(x + \dfrac{4\pi}{3}\right)$ ..............................

d) Expliciter $\cos(3x)$ en fonction de $\cos x$ ........................................

## Formules de duplication

### Calcul 8.5

En remarquant qu'on a $\frac{\pi}{4} = 2 \times \frac{\pi}{8}$, calculer :

a) $\cos \frac{\pi}{8}$ ..............................

b) $\sin \frac{\pi}{8}$ ..............................

### Calcul 8.6

a) Simplifier : $\frac{1 - \cos(2x)}{\sin(2x)}$ (avec $x \in \left]0, \frac{\pi}{2}\right[$) ..............................

b) Simplifier : $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$ (pour $x \in \left]0, \frac{\pi}{2}\right[$) ..............................

c) Expliciter $\cos(4x)$ en fonction de $\cos x$ ..............................

## Équations trigonométriques

### Calcul 8.7

Résoudre dans $[0, 2\pi]$, dans $[-\pi, \pi]$, puis dans $\mathbb{R}$ les équations suivantes :

a) $\cos x = \frac{1}{2}$ ..........

b) $\sin x = -\frac{\sqrt{3}}{2}$ .......

c) $\sin x = \cos \frac{2\pi}{3}$ ......

d) $\tan x = -1$ .........

e) $\cos^2 x = \frac{1}{2}$ .........

f) $|\tan x| = \frac{1}{\sqrt{3}}$ ......

g) $\cos(2x) = \frac{\sqrt{3}}{2}$ ......

h) $2\sin^2 x + \sin x - 1 = 0$

i) $\cos x = \cos \frac{\pi}{7}$ .......

j) $\sin x = \cos \frac{\pi}{7}$ .......

## Inéquations trigonométriques

### Calcul 8.8

Résoudre dans $[0, 2\pi]$, puis dans $[-\pi, \pi]$, les inéquations suivantes :

a) $\cos x \geqslant -\frac{\sqrt{2}}{2}$ ........

b) $\cos x \leqslant \cos \frac{\pi}{3}$ ........

c) $\sin x \leqslant \frac{1}{2}$ ............

d) $|\sin x| \leqslant \frac{1}{2}$ ...........

e) $\tan x \geqslant 1$ ............

f) $|\tan x| \geqslant 1$ ..........

g) $\cos\left(x - \frac{\pi}{4}\right) \geqslant 0$ ......

h) $\cos\left(2x - \frac{\pi}{4}\right) \geqslant 0$ .....

## Réponses mélangées

$\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ $\quad$ $\left\{\frac{\pi}{12} + k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{11\pi}{12} + k\pi, k \in \mathbb{Z}\right\}$ $\quad$ $\frac{\sqrt{6}+\sqrt{2}}{4}$ $\quad$ $\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, 2\pi\right]$

$\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$ $\quad$ $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\right\}$ $\quad$ $\left\{-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$ $\quad$ $0$ $\quad$ $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

$\left\{\frac{\pi}{7}, \frac{13\pi}{7}\right\}$ $\quad$ $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$ $\quad$ $8\cos^4 x - 8\cos^2 x + 1$ $\quad$ $\left\{\frac{\pi}{4} + k\frac{\pi}{2}, k \in \mathbb{Z}\right\}$ $\quad$ $\frac{\sqrt{6}-\sqrt{2}}{4}$

$\tan x$ $\quad$ $\left\{\frac{\pi}{4} + k\pi, k \in \mathbb{Z}\right\}$ $\quad$ $\left\{\frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{5\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\}$ $\quad$ $-\sin x$

$\left\{-\frac{\pi}{3}, \frac{\pi}{3}\right\}$ $\quad$ $4\cos^3 x - 3\cos x$ $\quad$ $2$ $\quad$ $\left\{-\frac{5\pi}{6}, -\frac{\pi}{6}\right\}$ $\quad$ $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$ $\quad$ $\frac{1}{\cos x}$

$\left\{\frac{\pi}{7} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{7} + 2k\pi, k \in \mathbb{Z}\right\}$ $\quad$ $-1-\sqrt{3}$ $\quad$ $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}, 2\pi\right]$

$\left[-\pi, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$ $\quad$ $\left[\frac{\pi}{4}, \frac{\pi}{2}\right[ \cup \left]\frac{\pi}{2}, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[ \cup \left]\frac{3\pi}{2}, \frac{7\pi}{4}\right]$ $\quad$ $-2\cos x$ $\quad$ $-\frac{1}{2}$

$\left\{\frac{\pi}{6} + k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{5\pi}{6} + k\pi, k \in \mathbb{Z}\right\}$ $\quad$ $2\cos x$ $\quad$ $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$ $\quad$ $\frac{\sqrt{6}-\sqrt{2}}{4}$

$\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[ \cup \left]-\frac{\pi}{2}, -\frac{\pi}{4}\right] \cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right[ \cup \left]\frac{\pi}{2}, \frac{3\pi}{4}\right]$ $\quad$ $0$ $\quad$ $\left\{-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$ $\quad$ $\left\{-\frac{\pi}{7}, \frac{\pi}{7}\right\}$

$0$ $\quad$ $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]$ $\quad$ $\left[-\pi, -\frac{\pi}{3}\right] \cup \left[\frac{\pi}{3}, \pi\right]$ $\quad$ $\left\{\frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{11\pi}{6} + 2k\pi, k \in \mathbb{Z}\right\}$

$-\sin x$ $\quad$ $\left\{\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}\right\}$ $\quad$ $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ $\quad$ $\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$ $\quad$ $\frac{\sqrt{2+\sqrt{2}}}{2}$

$\left[-\pi, -\frac{5\pi}{6}\right] \cup \left[-\frac{\pi}{6}, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$ $\quad$ $\left\{\frac{5\pi}{14}, \frac{9\pi}{14}\right\}$ $\quad$ $\left\{-\frac{11\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{11\pi}{12}\right\}$ $\quad$ $\left\{\frac{5\pi}{14}, \frac{9\pi}{14}\right\}$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ $\quad$ $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ $\quad$ $0$ $\quad$ $\left[0, \frac{3\pi}{8}\right] \cup \left[\frac{7\pi}{8}, \frac{11\pi}{8}\right] \cup \left[\frac{15\pi}{8}, 2\pi\right]$ $\quad$ $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[ \cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right[$ $\quad$ $\left\{\frac{\pi}{6} + k\frac{2\pi}{3}, k \in \mathbb{Z}\right\}$ $\quad$ $\left\{\frac{5\pi}{14} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{9\pi}{14} + 2k\pi, k \in \mathbb{Z}\right\}$

$\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$ $\quad$ $\left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}$ $\quad$ $\left[\frac{\pi}{4}, \frac{\pi}{2}\right[ \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[$ $\quad$ $\frac{\sqrt{2-\sqrt{2}}}{2}$

$\left[-\pi, -\frac{5\pi}{8}\right] \cup \left[-\frac{\pi}{8}, \frac{3\pi}{8}\right] \cup \left[\frac{7\pi}{8}, \pi\right]$ $\quad$ $\left\{\frac{-2\pi}{3}, \frac{-\pi}{3}\right\}$ $\quad$ $\left\{\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\}$

► Réponses et corrigés page 98

# Fiche nº 8. Trigonométrie

## Réponses

**8.1** a) ........ $\boxed{0}$

**8.1** b) ........ $\boxed{0}$

**8.1** c) ........ $\boxed{-1-\sqrt{3}}$

**8.1** d) ........ $\boxed{-\dfrac{1}{2}}$

**8.2** a) ........ $\boxed{0}$

**8.2** b) ........ $\boxed{-\sin x}$

**8.2** c) ........ $\boxed{2\cos x}$

**8.2** d) ........ $\boxed{-2\cos x}$

**8.3** a) ........ $\boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

**8.3** b) ........ $\boxed{\dfrac{\sqrt{6}+\sqrt{2}}{4}}$

**8.3** c) ........ $\boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

**8.3** d) ........ $\boxed{\dfrac{\sqrt{3}-1}{\sqrt{3}+1}}$

**8.4** a) ........ $\boxed{-\sin x}$

**8.4** b) ........ $\boxed{\dfrac{1}{\cos x}}$

**8.4** c) ........ $\boxed{0}$

**8.4** d) ........ $\boxed{4\cos^3 x - 3\cos x}$

**8.5** a) ........ $\boxed{\dfrac{\sqrt{2+\sqrt{2}}}{2}}$

**8.5** b) ........ $\boxed{\dfrac{\sqrt{2-\sqrt{2}}}{2}}$

**8.6** a) ........ $\boxed{\tan x}$

**8.6** b) ........ $\boxed{2}$

**8.6** c) ........ $\boxed{8\cos^4 x - 8\cos^2 x + 1}$

**8.7** a) ........ $\boxed{\left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right\}}$

**8.7** a) ........ $\boxed{\left\{-\dfrac{\pi}{3}, \dfrac{\pi}{3}\right\}}$

**8.7** a) ........ $\boxed{\left\{\dfrac{\pi}{3}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{-\dfrac{\pi}{3}+2k\pi, k\in\mathbb{Z}\right\}}$

**8.7** b) ........ $\boxed{\left\{\dfrac{4\pi}{3}, \dfrac{5\pi}{3}\right\}}$

**8.7** b) ........ $\boxed{\left\{\dfrac{-2\pi}{3}, \dfrac{-\pi}{3}\right\}}$

**8.7** b) ........ $\boxed{\left\{\dfrac{4\pi}{3}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{5\pi}{3}+2k\pi, k\in\mathbb{Z}\right\}}$

**8.7** c) ........ $\boxed{\left\{\dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}}$

**8.7** c) ........ $\boxed{\left\{-\dfrac{5\pi}{6}, -\dfrac{\pi}{6}\right\}}$

**8.7** c) ........ $\boxed{\left\{\dfrac{7\pi}{6}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{11\pi}{6}+2k\pi, k\in\mathbb{Z}\right\}}$

**8.7** d) ........ $\boxed{\left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}}$

**8.7** d) ........ $\boxed{\left\{-\dfrac{3\pi}{4}, \dfrac{\pi}{4}\right\}}$

**8.7** d) ........ $\boxed{\left\{\dfrac{\pi}{4}+k\pi, k\in\mathbb{Z}\right\}}$

**8.7** e) ........ $\boxed{\left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4} \dfrac{7\pi}{4}\right\}}$

**8.7** e) ........ $\boxed{\left\{-\dfrac{3\pi}{4}, -\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}\right\}}$

**8.7** e) ........ $\boxed{\left\{\dfrac{\pi}{4}+k\dfrac{\pi}{2}, k\in\mathbb{Z}\right\}}$

**8.7** f) ........ $\boxed{\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}}$

**8.7** f) ........ $\boxed{\left\{-\dfrac{5\pi}{6}, -\dfrac{\pi}{6}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}}$

**8.7** f) ........ $\boxed{\left\{\dfrac{\pi}{6}+k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{5\pi}{6}+k\pi, k\in\mathbb{Z}\right\}}$

**8.7** g) ........ $\boxed{\left\{\dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}\right\}}$

**8.7** g) ........ $\boxed{\left\{-\dfrac{11\pi}{12}, -\dfrac{\pi}{12}, \dfrac{\pi}{12}, \dfrac{11\pi}{12}\right\}}$

**8.7** g) ........ $\boxed{\left\{\dfrac{\pi}{12}+k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{11\pi}{12}+k\pi, k\in\mathbb{Z}\right\}}$

**8.7** h) ........ $\boxed{\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}\right\}}$

**8.7** h) ........ $\boxed{\left\{-\dfrac{\pi}{2}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}}$

**8.7** h) ........ $\left\{\frac{\pi}{6} + k\frac{2\pi}{3}, k \in \mathbb{Z}\right\}$

**8.7** i) ........ $\left\{\frac{\pi}{7}, \frac{13\pi}{7}\right\}$

**8.7** i) ........ $\left\{-\frac{\pi}{7}, \frac{\pi}{7}\right\}$

**8.7** i) ...... $\left\{\frac{\pi}{7} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{7} + 2k\pi, k \in \mathbb{Z}\right\}$

**8.7** j) ........ $\left\{\frac{5\pi}{14}, \frac{9\pi}{14}\right\}$

**8.7** j) ........ $\left\{\frac{5\pi}{14}, \frac{9\pi}{14}\right\}$

**8.7** j) ...... $\left\{\frac{5\pi}{14} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{9\pi}{14} + 2k\pi, k \in \mathbb{Z}\right\}$

**8.8** a) ........ $\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, 2\pi\right]$

**8.8** a) ........ $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]$

**8.8** b) ........ $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$

**8.8** b) ........ $\left[-\pi, -\frac{\pi}{3}\right] \cup \left[\frac{\pi}{3}, \pi\right]$

**8.8** c) ........ $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$

**8.8** c) ........ $\left[-\pi, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$

**8.8** d) ........ $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}, 2\pi\right]$

**8.8** d) ........ $\left[-\pi, -\frac{5\pi}{6}\right] \cup \left[-\frac{\pi}{6}, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$

**8.8** e) ........ $\left[\frac{\pi}{4}, \frac{\pi}{2}\right[ \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[$

**8.8** e) ........ $\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[ \cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right[$

**8.8** f) ..... $\left[\frac{\pi}{4}, \frac{\pi}{2}\right[ \cup \left]\frac{\pi}{2}, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[ \cup \left]\frac{3\pi}{2}, \frac{7\pi}{4}\right]$

**8.8** f) .... $\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[ \cup \left]-\frac{\pi}{2}, -\frac{\pi}{4}\right] \cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right[ \cup \left]\frac{\pi}{2}, \frac{3\pi}{4}\right]$

**8.8** g) ........ $\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$

**8.8** g) ........ $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$

**8.8** h) ........ $\left[0, \frac{3\pi}{8}\right] \cup \left[\frac{7\pi}{8}, \frac{11\pi}{8}\right] \cup \left[\frac{15\pi}{8}, 2\pi\right]$

**8.8** h) ........ $\left[-\pi, -\frac{5\pi}{8}\right] \cup \left[-\frac{\pi}{8}, \frac{3\pi}{8}\right] \cup \left[\frac{7\pi}{8}, \pi\right]$

## Corrigés

**8.3 b)** On peut utiliser $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ puis les formules d'addition.

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**8.4 b)** On a

$$\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \frac{\sin 2x \cos x - \cos 2x \sin x}{\sin x \cos x} = \frac{\sin(2x - x)}{\sin x \cos x} = \frac{1}{\cos x}.$$

On peut aussi faire cette simplification à l'aide des formules de duplication :

$$\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \frac{2 \sin x \cos x}{\sin x} - \frac{2\cos^2 x - 1}{\cos x} = \frac{1}{\cos x}$$

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**8.4 d)** On calcule

$$\begin{aligned}\cos(3x) = \cos(2x + x) = \cos(2x)\cos x - \sin(2x)\sin x &= (2\cos^2 x - 1)\cos x - 2\cos x \sin^2 x \\ &= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x) = 4\cos^3 x - 3\cos x.\end{aligned}$$

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**8.5 a)** On a $\cos\frac{\pi}{4} = 2\cos^2\frac{\pi}{8} - 1$ donc $\cos^2\frac{\pi}{8} = \frac{\frac{\sqrt{2}}{2} + 1}{2} = \frac{\sqrt{2} + 2}{4}$. De plus, $\cos\frac{\pi}{8} \geqslant 0$ donc $\cos\frac{\pi}{8} = \frac{\sqrt{2 + \sqrt{2}}}{2}$.

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**8.5 b)** On a $\sin^2\frac{\pi}{8} = 1 - \cos^2\frac{\pi}{8} = \frac{2 - \sqrt{2}}{4}$ et $\sin\frac{\pi}{8} \geqslant 0$ donc $\sin\frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$.

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**8.6** a) On a $\cos(2x) = 1 - 2\sin^2 x$ donc $\dfrac{1-\cos(2x)}{\sin(2x)} = \dfrac{2\sin^2 x}{2\sin x\cos x} = \tan x$.

**8.6** b) On a $\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x} = \dfrac{\sin 3x\cos x - \cos 3x\sin x}{\sin x\cos x} = \dfrac{\sin(3x-x)}{\sin x\cos x} = \dfrac{\sin(2x)}{\sin x} = \dfrac{2\sin x\cos x}{\sin x\cos x} = 2$.

**8.6** c) On a $\cos(4x) = 2\cos^2(2x) - 1 = 2(2\cos^2 x - 1)^2 - 1 = 8\cos^4 x - 8\cos^2 x + 1$.

**8.7** e) Cela revient à résoudre « $\cos x = \dfrac{\sqrt{2}}{2}$ ou $\cos x = -\dfrac{\sqrt{2}}{2}$ ».

**8.7** g) Si on résout avec $x \in [0, 2\pi]$, alors $t = 2x \in [0, 4\pi]$.

Or, dans $[0, 4\pi]$, on a $\cos t = \dfrac{\sqrt{3}}{2}$ pour $t \in \left\{\dfrac{\pi}{6}, \dfrac{11\pi}{6}, \dfrac{13\pi}{6}, \dfrac{23\pi}{6}\right\}$ et donc pour $x \in \left\{\dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}\right\}$.

**8.7** h) $\sin x$ est solution de l'équation de degré 2 : $2t^2 + t - 1 = 0$ dont les solutions sont $t = -1$ et $t = \dfrac{1}{2}$. Ainsi, les $x$ solutions sont les $x$ tels que $\sin x = -1$ ou $\sin x = \dfrac{1}{2}$.

**8.7** j) On a $\cos\dfrac{\pi}{7} = \sin\left(\dfrac{\pi}{2} - \dfrac{\pi}{7}\right) = \sin\dfrac{5\pi}{14}$. Finalement, on résout $\sin x = \sin\dfrac{5\pi}{14}$.

**8.8** d) Cela revient à résoudre $-\dfrac{1}{2} \leqslant \sin x \leqslant \dfrac{1}{2}$.

**8.8** f) On résout « $\tan x \geqslant 1$ ou $\tan x \leqslant -1$ ».

**8.8** g) Si $x \in [0, 2\pi]$, alors $t = x - \dfrac{\pi}{4} \in \left[-\dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}\right]$. On résout donc $\cos t \geqslant 0$ pour $t \in \left[-\dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}\right]$ ce qui donne $t \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right] \cup \left[\dfrac{3\pi}{2}, \dfrac{7\pi}{4}\right]$ et donc $x \in \left[0, \dfrac{3\pi}{4}\right] \cup \left[\dfrac{7\pi}{4}, 2\pi\right]$.

**8.8** h) Si $x \in [0, 2\pi]$, alors $t = 2x - \dfrac{\pi}{4} \in \left[-\dfrac{\pi}{4}, 4\pi - \dfrac{\pi}{4}\right]$. On résout donc $\cos t \geqslant 0$ pour $t \in \left[-\dfrac{\pi}{4}, 4\pi - \dfrac{\pi}{4}\right]$ ce qui donne $t \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right] \cup \left[\dfrac{3\pi}{2}, \dfrac{5\pi}{2}\right] \cup \left[\dfrac{7\pi}{2}, \dfrac{15\pi}{4}\right]$ puis $x \in \left[0, \dfrac{3\pi}{8}\right] \cup \left[\dfrac{7\pi}{8}, \dfrac{11\pi}{8}\right] \cup \left[\dfrac{15\pi}{8}, 2\pi\right]$.